Evaluating the Compressive Strength of Hail and Its Relationship to Freezer Iceballs

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Hailstone with cyclic growth ring structure and dry growth areas (cloudy) and wet growth areas (clear). Note the alternating growth types, which result from dry and wet growth conditions.

ABSTRACT

Hail damage to building envelopes is clearly on the rise. This is confirmed by insurance companies reporting that 2017 was the first year that hail damage has been the most expensive insured peril for real property claims in the United States. The causes for this unprecedented hail damage cost include more frequent damaging hailstorms, urban growth that puts more structures at risk, larger structures that have larger roofs, more expensive repairs due to higher materials and labor cost, and increased public awareness of the insureds' rights and responsibilities under the terms of their policies.

This article addresses the issue of damaging hail events and specifically addresses the compressive strength of hail. Hail impacts have typically been related to the size of the hailstones. Some investigators computed the kinetic energy of hail impacts and adjusted the hail's falling speed by the wind speed occurring during the hail event. ASTM E822-92 provides numerical methods for adjusting hail falling speed based upon the hail's terminal velocity and wind speed. Many people have the experience of observing hail in the form of soft slush balls, while some hail is seemingly as hard as a ball bearing.

More than 875 natural hailstones were analyzed for size, density, compressive strength (F_o), and compressive stress (σ_c) by the Insurance Institute of Business and Home Safety (IBHS). These data were analyzed with a variety of methods, and analysis shows a clear trend in the increase of compressive strength and compressive strength uniformity with increases in hail diameter by size bin. In addition, data were considered by hail size bins, beginning with ½ through 2½ inches in ¼-in. group bins. These data (F_o and σ_c) were regressed with an exponential equation with $R^2 = 0.97$ and 0.95, respectively. The data show that as diameter increases, so does F_o and its uniformity; however, σ_c will decrease with diameter increase, while uniformity of σ_c increases with increased diameter. The hail F_o and σ_c population distribution was developed for each hail size bin. The kinetic energy (KE) was computed for each hail size bin.

Natural hail compressive strength from 875 natural hailstones from the IBHS were compared to the compressive strength of 585 iceballs, and their mathematical relationship was developed for size bins ½ though 2½ inches in ¼-in. increments. The measurement systems and testing environment were carefully controlled and similar to the systems used by IBHS in collecting their data. The data collections system (caliper measurement) was supplemented with a liquid displacement (Archimedes method) for computing volume, density, and cross-sectional area. These measurements are also used to compute the compressive stress (σ_c).

Of fundamental significance is the fact that the IBHS hailstones were modeled after oblate spheroids, where the iceballs, cast in plastic molds, are near-perfect spheres. The cross-sectional areas of the two shapes were computed. A uniaxial compressive force was used to measure the F_o of iceballs with a hydropneumatics press, whereas IBHS used a hand-operated modified compression clamp. The testing environment temperature was a constant 28°F (-2°C), and iceballs were made from distilled water and were refrigerated to -16°F (-27°C) for 24 hours prior to testing.

INTRODUCTION

Traditionally, hail is evaluated based upon size; however, size alone does not tell the whole story. Some roof and building envelope investigators compute hail's kinetic energy (KE) (*Equation 1*) that considers both the hail weight and falling speed.

1. KE = ½ m V²

Where:
KE = Kinetic energy (lbf or Joules)
m = mass of the hailstone (lb. or kg)
V = Velocity of the falling hailstone at impact (mph or m/s)

Some investigators have observed that hailstorms frequently occur with wind that will affect the falling speed and, thus, KE. Wind causes the speed a hailstone is falling at to increase (*Equation 2*) and will increase KE (ASTM E822-92). Since the KE velocity term is squared, increased KE due to velocity is exponential, making KE sensitive to changes in velocity.

2. FS = √TV²+WS²

Where:
FS = Failing speed of hailstone at time of impact = V
TV = Terminal velocity of hailstone at time of impact (mph or m/s)
WS = Wind speed during the hail event (mph or m/s)

Since KE is sensitive to changes in velocity, it is necessary to understand how fast hail is moving at the time of impact. Hail falling speed is affected by many conditions, including terminal velocity and wind speed. Wind speed also affects the angle the hail is falling. Hail KE must be resolved into direct and shear energy at impact. This energy calculation requires vector analysis that accounts for both direction and magnitude to resolve the KE into direct and shear forces.

HISTORY OF HAIL COMPRESSIVE STRENGTH TESTING

Early work on hail compressive strength focused on hailstone air content. Several researchers, including Browning (1968), List (1972 and 1973), Knight (1970, 1974, 1978, 1983, and 2001), and Heymsfield (1983, 2014[a], and 2014[b]) investigated air content of hailstones and their falling behavior. Fewer researchers have investigated the compressive strength of hail. Of the 316 references used in this work, only 15 (less than 5%) discussed the compressive strength of hail, five dealt with strain rates of ice cylinders that were not hailstones at all, and only three (<1%) papers specifically addressed the compressive strength of hailstones.

The term "hardness" typically refers to a surface condition and not the entire structure. The verbiage used to describe hail has been confusing. In terms of σ_c, we use the F_o divided by the cross-sectional area of the average diameter. This is the σ reported in most materials testing literature in stress/strain relationships. Strain (ε) is the distance by which the sample is compressed by σ_c.

Many hailstones are damaged upon impact. In short, the investigator must be on the spot and prepared to perform the investigation before the hail has changed from its impact condition. Even in vehicles equipped with computers and radar software, it is not easy to get to the right spot at the right time. Often, ideal locations for a hail collection are off paved roads that are not accessible to storm chase vehicles—especially ones pulling a mobile lab. The research mobile lab operates at 28°F, and samples are refrigerated until tested. Hailstone temperature is the first item measured, and testing is done as soon after collection as safely possible. Samples stored for long will change from their antecedent condition. Most commercial freezer units will produce temperatures between -24 and -4°F. Compressive strength of ice increases dramatically as the temperature decreases from -10 to -50°C (14 to -58°F). Ice surface hardness can increase 240% from a temperature of 0 to -80°C (32 to -112°F) (Chaplin, 2016). This work requires the investigator to take the lab to the field. Chasing storms is neither easy nor cheap and has numerous risks.

Giammanco et al. (2015) used a pis-tol grip clamp with attached load cell to measure hailstone compressive strength. Measurements showed hail compressive strength ranged from as low as 0.19 to as high as 490.64 lbf. The temperature was not reported.

Butkovich (1958) used a Brinell hardness tester to measure the hardness of the surface of single ice crystals. These data showed that the Brinell hardness number varied by temperature by more than 300% in the temperature range from -15°C to -50°C (in other words, the colder the temperature, the harder the ice crystals).

Note that Brinell's tests were done on the surface; therefore, hardness would be an appropriate term to describe their results. Earlier work by Barnes (1928), Moor (1940), and Blackwelder (1940) measured the scratch hardness of the surface of ice crystals using the Moh's hardness scale. The obvious difference between the work from Giammanco and Butkovich, Barnes, Moor, and Blackwelder is that the last four researchers only tested the surface conditions of hailstones and not the compressive strength of the entire stone.

Only the work by Giammanco et al. (2015) addresses the compressive strength of the whole hailstone. For the work to be specific to what causes hailstones to damage roofs and other elements of building envelopes, the whole stone must be considered, not just its surface condition.

Most hail impact testing is done using iceballs. Many building envelope products are not even tested with iceballs, with investigators opting for ASTM D3746, FM 4470, or UL 2218—all of which rely upon steel balls or steel missiles impacting building envelope materials. Steel projectiles do not accurately represent ice due to the difference in surface hardness and terminal velocity of the different materials. Steel balls are denser than ice and are more uniform; they do not replicate the types of damage caused by hailstones. Iceballs are not the same as natural hailstones; they are similar in surface hardness and terminal velocity, but typically denser with higher F_o.

ASTM E822-92 states "no direct relationship has been established between the effect of impact of iceballs and hailstones." (UL 2218 contains similar language.) This paper offers such a relationship that will allow new insights into how hail impacts building envelope products and other materials. This will allow manufacturers a direct comparison of iceball testing to natural

hailstones—something that no "steel ball" test can do. It would seem logical that at some point, additional increases in F_o do not cause additional damage. This work expands upon the original dataset collected by Giammanco et al. (2015). This dataset allows direct comparison between natural hailstones and man-made iceballs.

THE LANGUAGE PROBLEM

One of the difficulties in investigating the material science of hail is the words used. To reduce confusion and provide clarity, we have chosen to use the verbiage for the various disciplines used in this investigation. This investigation utilized meteorology, materials science, engineering, and statistics. This effort in clarification extends to abbreviations and symbols too. A few notable words/symbols and their meanings include:

- Hardness – toughness of the surface of the object
- Compressive strength (F_o) – applied load at the point of rupture or failure
- Compressive stress (σ_c) – F_o divided by the cross-sectional area
- Ultimate stress – stress at the point of rupture or failure
- Strain (ε) – distance the sample is compressed during testing
- Uniaxial load – load applied along one axis in one direction
- Standard deviation (SD) – in most scientific literature, the standard deviation uses the symbol σ; however, since we are discussing stress, which also uses the same symbol, we use the abbreviation SD for standard deviation.

FRACTURE MECHANICS

The topic of hailstone and iceball fracture mechanics is covered in some detail in Phelps (2018c). The study of fracture mechanics of hailstones and iceballs is complicated by shape, surface roughness, and material types. The shape problem is due to spherical or oblate objects that are not typically used in materials testing.

The typical shape for materials tests tends to be cylinders with a ratio of the diameter (d) to length, 2d. The stress/strain curve created by plotting σ_c against strain (ε) has numerous points defined by the shape or change of slope of the curve. Hailstones and iceballs are affected by hoop stress, which is not typically present in traditional testing shapes. Most ice strain rate literature is reported for ice cylinders and not spherical or oblate shapes. The shape makes hailstone and iceball tests unique among materials testing professionals.

The other issue with analyzing stress/strain relationships is the ice itself. Many of the stress/strain diagrams found in the literature are from materials that are homogeneous and isotropic. Ice is neither—especially natural hailstones. Ice grows on three axes and the perpendicular (vertical) axis can have a Youngs modulus 30% higher than the two horizontal axes (Schulson, 1999). Since natural hailstones have varying amounts of ice, air, and liquid water, they are seldom well behaved in stress/strain tests, and this is further complicated by the fact that the ratio of ice, air, and water can change with each layer or region within the hailstone. Hailstones may have many ice crystals, each of which has its own separate set of axes that are randomly positioned. Not all hailstones have layered or cyclic construction (Butkovich, 1958).

Practically all ice on this planet is type Ih or hexagonal ice (Chaplin, 2012). Hexagonal ice is so called due to its six equivalent prism faces. Hexagons typically have internal angles totaling 720°. The angles of hexagonal ice (ice Ih) are due to the bond angle of the hydrogen=oxygen bonds from which they are made. The weakness of ice comes not only from the bond but from the discontinuities of entrained/trapped air and liquid water within the ice voids. Hexagonal ice has triple points with liquid and gaseous water. A triple point is a temperature and pressure combination at which liquid water, solid ice, and water vapor can coexist in stable equilibrium (0.01 °C, partial pressure of 611.657 pascals, and others) (Chaplin, 2012).

WEATHER DATA

Hail occurs in many storm types, with the largest hail typically occurring in a high precipitation supercell. The National Weather Service (NWS) uses hail size as one of the two conditions that trigger severe storm warnings. Hail 1 inch in diameter or wind surface speeds above 58 mph will trigger a severe storm warning. In Canada, hail size of 20 mm (0.79 in.) or wind greater than 90 kph per hour (56 mph) will trigger a severe weather warning.

Hailstone diameters vary from storm to storm but also within a storm. Not all hail from a single storm comes from the same cloud cell, and even those that do will still have a size distribution. Since many hailstorms have distributions—with the higher population occurring with smaller-size hail and the lower population from larger, more-damaging hail—it is more likely than not that NWS or other spot-

	½ in.	¾ in.	1 in.	1¼ in.	1½ in.	1¾ in.	2 in.	2¼ in.	2½ in.
Typical TV (mph)	36.76	45.02	51.98	58.12	63.66	68.77	73.51	77.97	82.19
Typical KE (joules)	0.13	0.67	2.12	5.18	10.75	19.92	33.98	54.43	82.96

Table 1 – Size of hail bins and typical TV and KE.

Count	134	250	230	117	72	43	21	8	4
Size Bin	< ½"	< ¾"	< 1"	<1¼"	< 1½"	<1¾"	<2"	<2¼"	< 2½"
Max. (lbf)	254.92	314.64	366.89	266.51	490.64	200.71	201.10	237.48	341.64
Min. (lbf)	0.19	0.84	2.31	4.01	4.01	17.18	45.85	70.3	82.17
Avg. (lbf)	28.28	29.476	37.84	55.11	66.67	91.710	102.46	106.5	189.61
Variance	2323.3	1257.3	1695.	1300.7	2964.7	1786.8	1371.9	2907.5	12228.42
SD	48.20	35.459	41.17	36.07	54.449	42.271	37.04	53.921	110.582
Avg. Diam. (in)	0.405	0.6283	0.852	1.103	1.3601	1.6096	1.825	2.0792	2.56282

Table 2 – Natural hail compressive strength (lb_f) bin information.

ters will report the most frequently observed hail size. NWS does not train spotters to search for the largest hailstone, only to report the largest one they collect. NWS-trained spotters are instructed to:

1. Avoid harm's way and personal injury
2. Collect whole undamaged hailstones
3. Report the diameter of the largest hailstone collected by way of comparison to common objects

The NWS provides no training to collect any specific or relevant-size hailstones. Which hailstone a spotter selects is up to the individual. The relevant size (measured or other description such as golf ball or quarter) is what is reported by the spotter. Not all spotters are NWS trained.

HAIL TESTING

The IBHS collected hail samples from several storms over a seven-year period. Most locations were in the U.S. Midwest. The IBHS collected all sizes of hail and did not limit samples by size, condition, or fitness for any use. Diameters were measured with calipers on three axes, and stones were weighed before they were crushed. The IBHS used a hand-operated clamp to compress the stone against a load cell. Temperature was not recorded. This process did not permit controlling the speed of application of force (strain rate). *Table 1* shows typical terminal velocities and resulting KE for a variety of hail diameters.

IBHS used a hand-operated pistol grip clamp to apply uniaxial force to the hailstones. The strain rate was not well controlled in their investigation, and differences would occur with different operators, arm strength, and fatigue. IBHS reports strain rates around 10^{-1} s^{-1} (Giammanco, 2015). In this investigation, the iceballs were compressed using a hydro-pneumatic press that provided a similar strain rate to that found by IBHS.

IBHS used a 2.2-kN (500-lb.) load cell with a Wheatstone bridge, signal conditioner (48-kHz), and LabView software. In similar fashion, we used an 8.8-kN (2,000-lb.) Omega Engineering load cell with function generator, Omega Engineering Data Acquisition (DAQ) module (collect 50 and report 10-kHz), and TracerDAC Pro software. The load cell and other instruments were calibrated in conformance with ISO 17025 and possess NIST traceability. This method of applying compressive load to the iceballs is also uniaxial (load applied from only one side along a single axis). The cold lab-operating temperature was about -2°C (28°F).

OBSERVED DATA

The IBHS data were collected from about 33 hail days from 2012 through 2017 in Texas, Oklahoma, Kansas, Nebraska, South Dakota, and Minnesota. A total of about 3270 stones were collected. Those records that were incomplete or had erroneous data, such as negative applied load values, were removed from the analysis. This left 879 stones with sufficiently complete data records, with associated strength tests that would permit further analysis and comparison to our dataset of iceballs.

To understand these data, they were considered in numerous ways:

- Storm data/location
- Hail diameter (measured on three locations or axes)
- Mass (weight)
- Hail compressive strength

The data were divided into bins by diameter, similar to NWS's. Since the natural hailstones collected by IBHS tended to be oblate, they computed the cross-sectional area as the longest axis $x_1/2$ and the shortest axis $y/2$, following Knight (1986). The cross-sectional area of this shape was computed as $A_{cross-section} = \prod ab$, with a being $x_1/2$ and b as $y/2$ (Giammanco, 2015). The average diameter of iceballs was computed as the numerical average of the three measurements. Cross-sectional areas were computed using these same average diameters ($A=\prod r^2$) where the radius is half the diameter; as noted by Giammanco (2015), "laboratory stones are nearly perfect spheres."

The ¼-in. increment was used because it includes 1-, 1¼-, and 2-in. bins. The 1-in. bin is included because NWS identifies 1 in. as the threshold for severe hail, 1¼ in. is a frequent diameter damage threshold in the published literature, and 2 in. is identified by the NWS as "significant hail." The ¼-in. increment includes all these bins and results in nine bins that provide an acceptable sample distribution range. Starting with ½ in., they increased in ¼-in. increments, arriving at <½, <¾, <1, <1¼, <1½, <1¾, <2, <2¼, and <2½ in. These nine bins form the basis from which other analyses were performed. The <1½ in. notation is used for the size of bin as a label and not a mathematical operation. *Table 2* shows size bins, number of stones in each bin, SD,
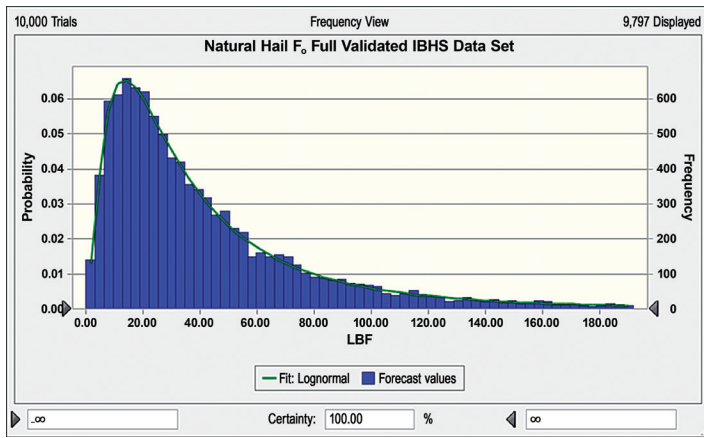
Figure 1 – Hail F_o in units of lb_f for all validated data from IBHS.

maximum, minimum, and average values for compressive strength (lbs. force, lbf) and average diameter (in.).

The F_o in lb_f for natural hail varies within and between size groups. *Figure 1* shows the range and distribution of hailstones' F_o collect by IBHS.

the same distribution type.

Hailstones were grouped by diameter because that is how hail size is reported by the NWS and other weather services. The incremental increase (¼ in.) was chosen because it allowed us to display the data for 1-in. hail (NWS severe weather condition),

The size and density of hail was also measured by IBHS and the data distribution displayed with Crystal Ball by Oracle. *Figures 2A* and *2B* show the probability distribution of average diameter of the hail samples collected by IBHS. Storms are different, and not all storms or hail diameters will have

2-in. (NWS significant hail size), and 1¼ in., which is a hail diameter that is common to the damage threshold size for many building envelope products (Marshall et al., 2002, 2004, and 2010; Koontz, 1991 and 2000; Cullen, 1997; and Greenfeld, 1969). The average compressive strength for each bin was analyzed and plotted, along with the range of compressive strength, number of samples in each bin, and the average diameter for each bin. *Figure 3* shows the results of this grouping, along with the curve fit and associated R^2 for the average compressive strength and average diameter for each size bin. *Figure 4* shows the range of F_o values. As the hail diameter increases, F_o becomes more uniform. The average F_o increases with diameter, too. These trends show that as hail grows larger, it tends to be stronger and more uniform. One probable reason for the increase in F_o is that it is proportional with the increase in diameter; the hailstone structure type is also a prob-
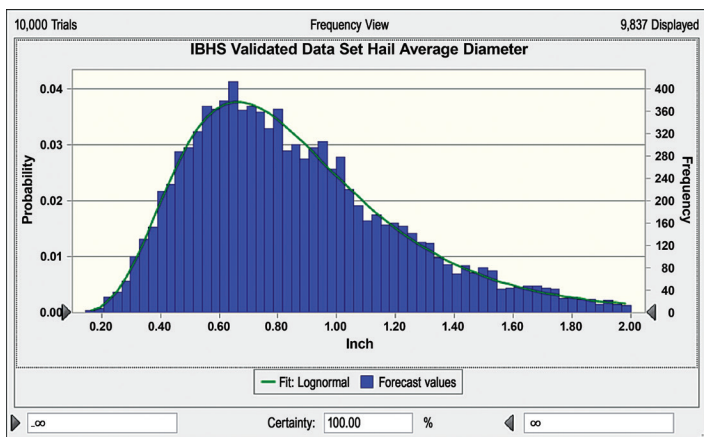

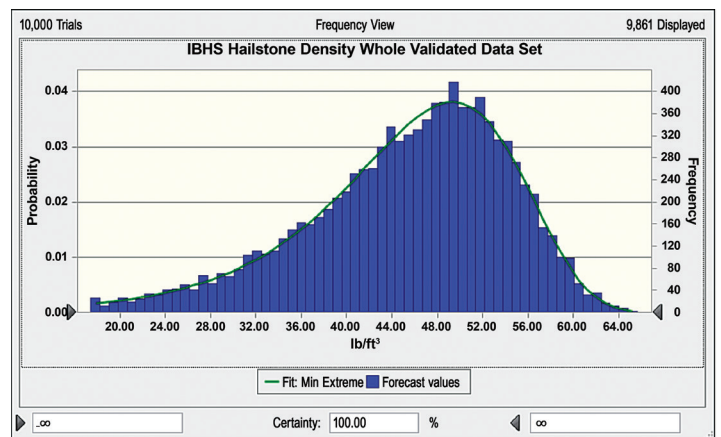*Figure 2A – Hail average diameter distribution from IBHS.*

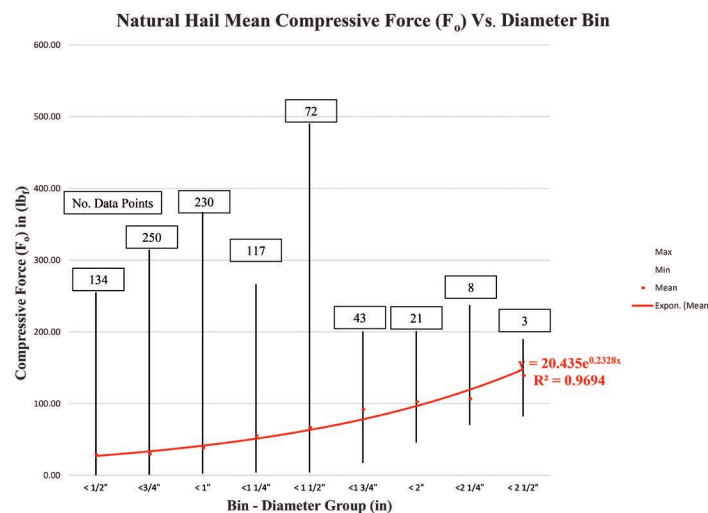*Figure 2B – Hail density distribution from IBHS.*

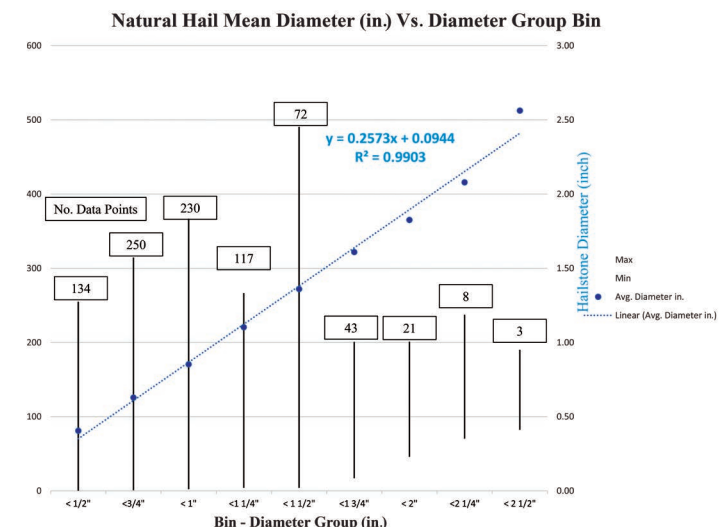*Figure 3 – Natural hail maximum, minimum, and mean F_o relationship to diameter bin.*

Figure 4 – Natural hail average diameter for each group bin.

NATURAL HAIL COMPRESSIVE STRESS BY SIZE GROUP

$y = 0.2573x + 0.0944$
$R^2 = 0.9903$

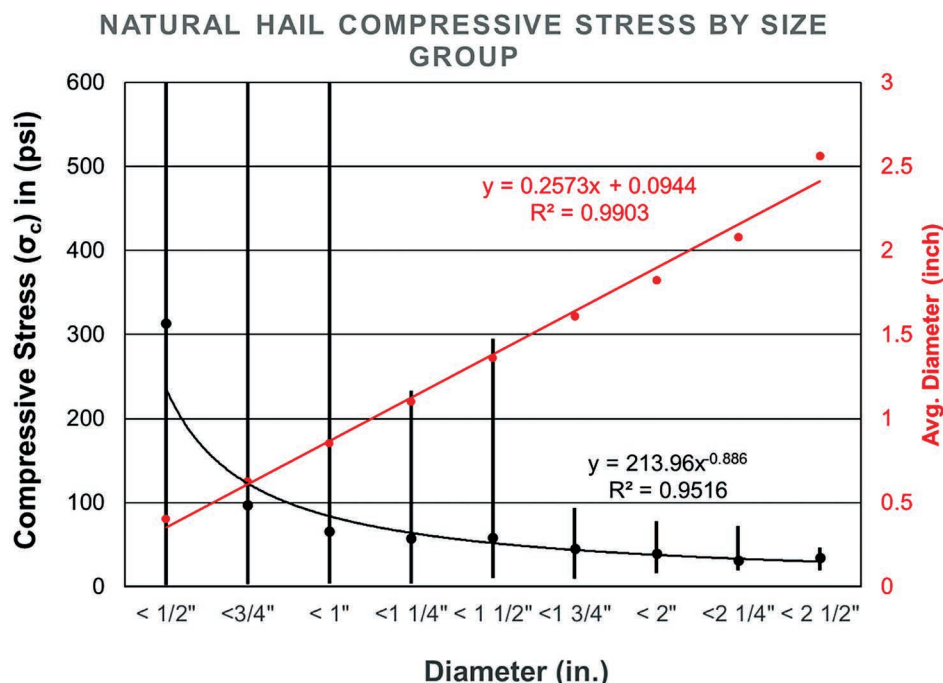$y = 213.96x^{-0.886}$
$R^2 = 0.9516$

Figure 5 – Relationship between hail compressive stress and hail diameter bin.

able reason. Large hail is sometimes formed by cyclic growth produced by cycling up and down in the updraft/downdrafts of the cloud cell. With each cycle, growth rings form inside the hailstone. All hail growth varies between dry and wet growth types; dry growth freezes on contact, while it takes longer for wet growth to freeze, allowing dissolved gases time to escape.

The cross-sectional area was computed using the average of the three axis lengths. This measurement was chosen to provide a uniform basis for the cross-sectional area calculation as shown in *Equation 3*.

3. $A = \Pi * r^2$

Where:
A = cross-sectional area (sq. in.)
Π = mathematical constant, general taken as 3.14159
r = radius as the average diameter/2

Compressive stress as shown in *Equation 4* uses the measured F_o and the samples' calculated cross-sectional area. Since the natural hailstones are assumed to be oblate spheroids, and the freezer iceballs are near perfect spheres, their cross-sectional areas are computed according to their shape and diameter.

4. $\sigma_c = F_o/A$

Where:
A = cross-sectional area (sq. in.)
σ_c = uniaxial compressive stress (psi)
F_o = applied compressive force (lbf)

Another interesting observation is that the spread of the data for uniaxial σ_c and peak F_o is reduced with increasing diameter. The reason for the reduced variability of σ_c and F_o is due to increased diameter with each bin. This is especially true for those diameter bins greater than 1 in. The identified trends with both peak σ_c and uniaxial F_o with diameter are both statistically significant, with the fitted curves accounting for 96.94% of the variance in *Figure 3* and 95.16% of the variance in *Figure 5*. *Figures 3* and *4* have only eight and three data points for bins <2¼ in. and <2½ in. respectively; these two data bins should be used with caution.

Since the hail grouping is based upon diameter and they are in ½-inch bins, this limits the variability to bin size. For example, the variance between bins may be larger than the variance within bins. This is evident in both *Figures 3* and *5* by the length (height) of the vertical lines that show the high and low for each bin. *Figure 5* shows only that portion of the high values below 600 psi.

Figure 6 shows the relationship between σ_c, SD, and hail size. These data confirm the large variability in compressive strength for small (<½ in. bin) hail whose SD is over 580

Count	65	61	36	31	62	63	60	35	53
Size	<½"	<¾"	<1"	<1¼"	<1½"	<1¾"	< 2"	<2¼"	<2½"
Max (lbf)	76.47	247.30	258.30	270.80	359.80	541.20	430.80	708.70	892.90
Min (lbf)	5.16	11.19	38.81	47.30	65.56	76.76	87.97	169.30	133.80
Avg. (lbf)	27.68	40.65	100.30	127.78	149.24	211.62	258.07	392.54	428.50
Variance	202.41	1243.09	3532.81	2880.26	3493.36	7549.64	6280.41	14414.80	23275.28
SD	14.23	35.26	59.44	53.67	59.10	86.89	79.25	120.06	152.56
Diameter									
Max (in.)	0.49	0.74	0.99	1.24	1.49	1.74	1.99	2.24	2.49
Min (in.)	0.38	0.50	0.75	1.01	1.25	1.50	1.75	2.00	2.25
Mean (in.)	0.45	0.59	0.85	1.15	1.32	1.64	1.83	2.14	2.38
Variance	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01
SD	0.03	0.08	0.06	0.07	0.06	0.07	0.06	0.07	0.07

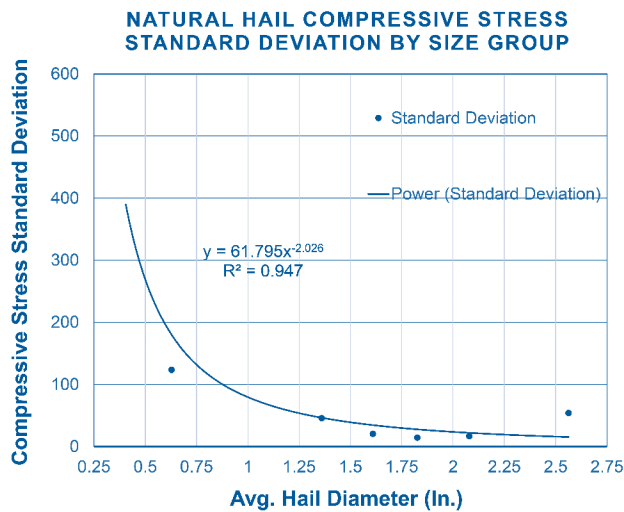*Table 3 – Iceball bin information.*

Figure 6 – Relationship between compressive stress SD and average diameter by bin in ¼-in. increments.
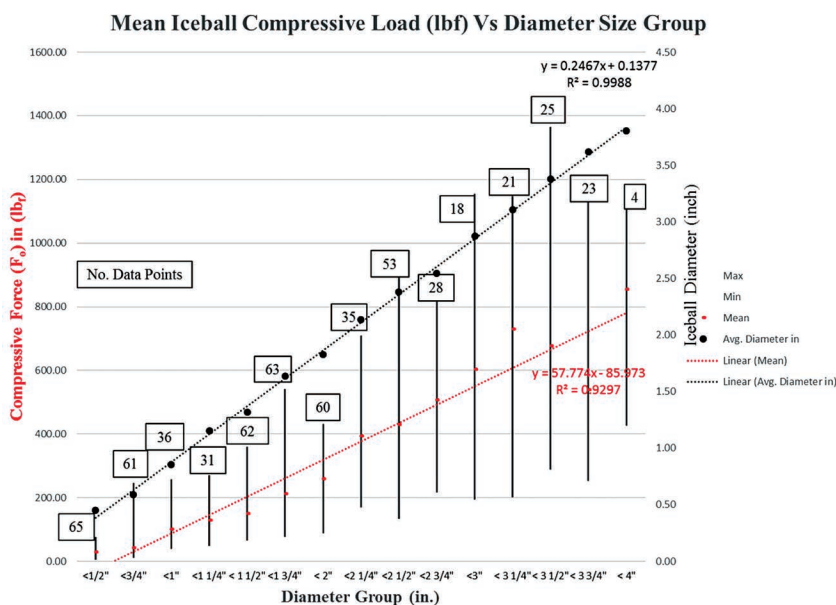

Figure 7 – Iceball F_o at failure by diameter group bin.

(134 samples); whereas 1-in. hail has a SD of only 36 (230 samples), the SD for the largest-hail-diameter bin was 12 (3 samples). *Figure 6* shows a clear trend that as hail diameter increases, the SD decreases. This is further evidence that small-hail (<1 in.) F_o is much more variable than larger hail. Our hypothesis is that crack length must increase as hail grows larger; hail structure, with cyclic growth, results in compressive load increases, and the variance of the data is reduced. This hypothesis is supported by the reduction of the SD with increasing diameter.

RELATIONSHIP TO FREEZER ICEBALLS

The compressive strength of iceballs has long been debated, and comparisons of hail compressive strength to the compressive strength of iceballs have been anecdotal while no real numerical comparisons have been made. This prompted the American Society of Testing and Materials, now ASTM International, to conclude in their standard E822-92, *Standard Practice for Determining Resistance of Solar Collector Covers to Hail by Impact With Propelled Ice Balls*, that "no direct relationship has been established between the effect of impact of ice balls and hailstones." We compared F_o in lbf and σ_c in psi for both natural hailstones and iceballs. This was done by grouping the two types into ¼-in. size bins beginning with ½ through 2½ in. The number of samples for natural hail and iceballs within each bin are included in *Tables 2 and 3*.

To compare natural hail to iceballs, the data for each were graphed. The graphs of the natural hail data are shown in *Figures 3, 5,* and *6*. Similar graphs for iceballs are shown in *Figures 7, 8,* and *9*.

Figure 7 shows the range of the bin compressive strength, average compressive strength, average diameter, and number of samples within each bin. The R^2 for both average diameter and compressive strength are significant.

Figure 8 shows the same data set as *Figure 7*; however, the compressive stress (compressive load divided by cross-sectional area) is graphed. The cross-sectional area is increasing faster than the applied load; therefore, we have a decrease in psi with increasing bin average diameter.

Figure 7 shows the compressive load (lbf) for bin

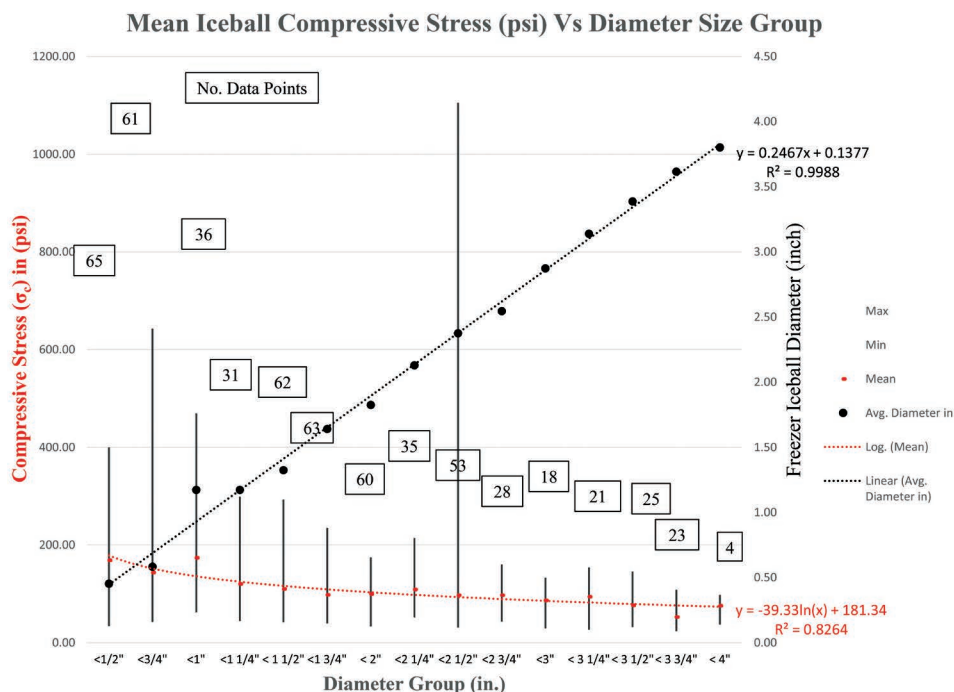## Mean Iceball Compressive Stress (psi) Vs Diameter Size Group

Figure 8 – Compressive stress (psi) at failure for iceballs by diameter group bin.

size for natural hail and iceballs respectively; similarly, *Figures 5* and *8* show the compressive stress (psi) for natural hail and iceballs. *Figures 6* and *9* show the relationship between standard deviation by size bin for natural hail and iceballs, respectively.

Iceballs are cast in a plastic/rubber mold. As one might expect, they are more uniform in size and construction than natural hailstones. The SD for iceballs was 106 to 33; the SD for natural hailstones was 582 to 12. Both natural hailstones and iceballs show more variation for small size (½ in.) than large size (2½ in.). The number of samples for the larger-size bins is minimal and should be used with caution. In *Figures 10* and *11* we see the relationships between natural hailstones and iceballs as F_o in lb_f and σ_c in psi, respectively.

Both the models for compressive load and compressive stress R^2 values are significant for natural systems (>0.6).

Figures 12A and *12B* show the plots

of mean hail measured to modeled F_o values and the mean hail measured to modeled σ_c values. These plots display how well the model equations from *Figures 10* and *11* compare to the measured data.

The reason for this variability is that the natural process of hail creation produces more variability than distilled water iceballs frozen in a plastic mold. Additional details on hail content of air, liquid water, and ice with engineering and materials properties on this subject are found in Phelps (2018b).

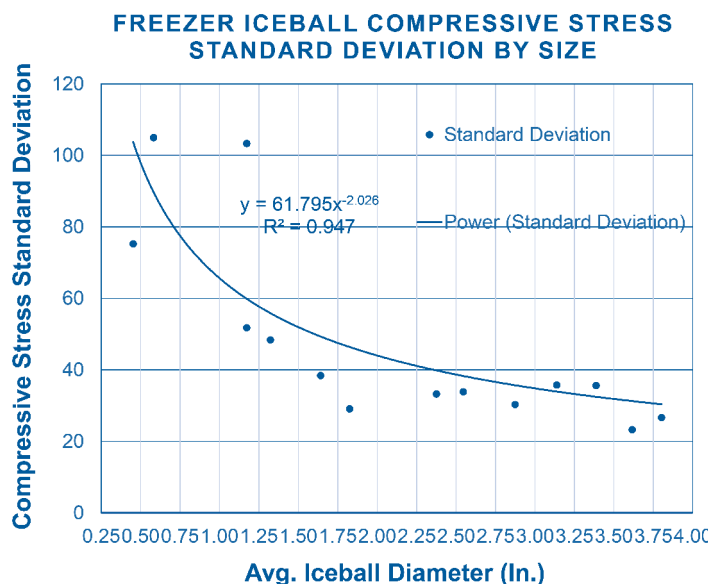Iceballs are frozen from the outside in;

FREEZER ICEBALL COMPRESSIVE STRESS STANDARD DEVIATION BY SIZE

$y = 61.795x^{-2.026}$
$R^2 = 0.947$

Figure 9 – Standard deviations of compressive stress iceballs by average diameter.

the exterior freezes first, and the inside, last. Natural hail is just the opposite: it freezes from the inside first, and then adds layers as it continues to grow. The "layering growth" may add uniformity to natural hail. The causes of variability in hail and iceballs include some of the same mechanics; however, hail also has the added complexity of different growth types (wet and dry) and structural types (static, cyclic, impact, and conglomerate).

MONTE CARLO ANALYSIS AND ANOVA

Monte Carlo methods utilize the probability distribution of a data set to estimate the expected values for many observations. Most Monte Carlo simulations are performed thousands of times such that the results can be used to estimate the probability of an outcome.

The distributions for the average F_o and σ_c for natural hailstones and iceballs are shown in *Figures 13A, B, C,* and *D.* These distribution graphs are helpful to understand the probability of a specified outcome. The distributions for F_o and σ_c look similar; they are the same distribution type (lognormal) for the entire data set. The distribution types vary by diameter.

With an understanding of the probability distributions of the natural hail and iceball F_o and σ_c, we can now compare natural hail F_o and σ_c to iceballs. This is a necessary step in assessing the comparison of natural hail to iceballs. *Figures 14A* and *14B* show the probability of F_o or σ_c of natural hail being equal to or greater than iceballs.

From *Figures 14A* and *14B*, we see that natural hail has a probability of 13.51% and 34.5% of F_o and σ_c being equal to or greater than iceballs. Generally, iceballs are stronger than natural hail.

Another method of assessing the variability between natural hail and iceballs modeled as natural hail is analysis of variance (ANOVA). ANOVA is a vetted statistical test that is often used to test the means of one data set against another. ANOVA is an exceptional tool for testing hypotheses. In this case, the null hypothesis is that iceballs modeled as natural hailstones are the same value as the natural hailstones; in other words, the averages for the two are the same.

In the F_o ANOVA, the null hypothesis is that the means (between the measured and modeled F_o values) are the same. The ANOVA in *Table 4* confirms that our iceball model for natural hail F_o is appropriate and fit for use in the manner shown.

In the σ_c ANOVA, the null hypothesis is that the means (between the measured and modeled σ_c values) are the same. The σ_c ANOVA in *Table 5* confirms that our iceball model for natural hail σ_c is appropriate and fit for use in the manner shown.

CONCLUSIONS

In weather systems, hail size is measured by diameter. It is logical and reliable to base this investigation upon size bins and, for those bins, to include 1-, 1¼-, and 2-in. sizes. Giant-size hail may be more dangerous than the sizes reported here; however, it is also rarer.

The compressive strength resistance of hail is highly variable—especially with small-diameter hail—and tends to become less variable as diameter increases. It is logical to reason that compressive strength increases with diameter due to increased

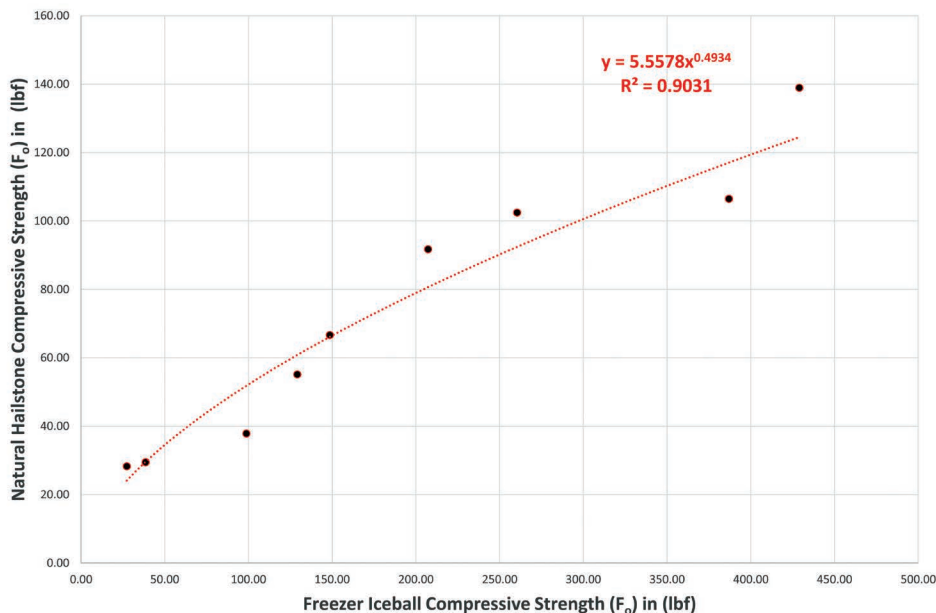**Compressive Strength (F_o) Natural Hailstone Direct Relationship To Freezer Iceballs**

$y = 5.5578x^{0.4934}$
$R^2 = 0.9031$

Figure 10 – F_o in lb_f natural hailstones direct relationship to iceballs of the same diameter.

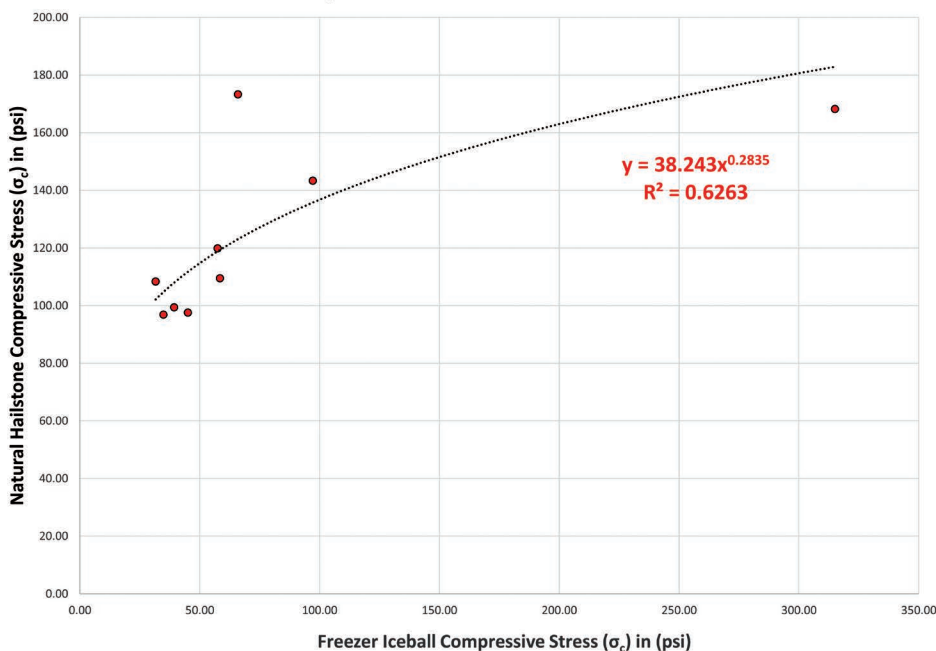**Compressive Stress (σ_c) Natural Hail Direct Relationship To Freezer Iceballs**

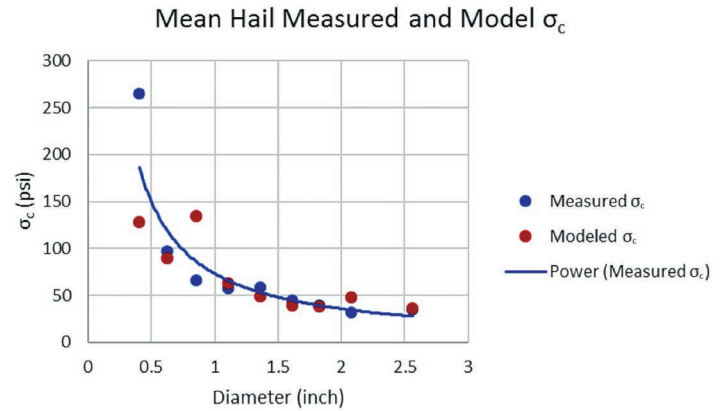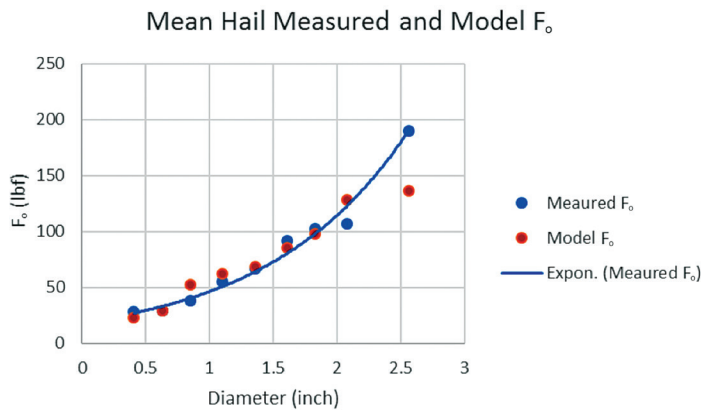$y = 38.243x^{0.2835}$
$R^2 = 0.6263$

Figure 11 – σ_c in psi natural hailstones direct relationship to iceballs.

Figures 12A and 12B – Side-by-side comparison of model results to measured data. These data confirm the goodness of the model fit to the measured data for both F_o and σ_c.

crack length, and variability is reduced with hail structure that is produced within strong, high-precipitation updrafts that promote cyclic wet growth.

The data collected and described by Giammanco et al. (2015) provides reliable evidence of the F_o of natural hailstones. Observations of iceballs in this investigation provide reliable evidence that F_o is stronger

than natural hail. Iceballs are often used in impact tests. Both hail and iceball data sets were collected using similar instruments and methods. The numerical analysis methods of the two data sets were the same.

The relationship between hail and iceballs allows investigators, engineers, meteorologists, manufacturers, testing labs, and educators to mathematically relate natural

hailstones to iceballs. This work provides new insights into how we assess and test hail impacts on building materials and other materials and provides manufacturers with a direct comparison of iceball testing to natural hailstones, making possible standard methods based upon iceball impact effects.
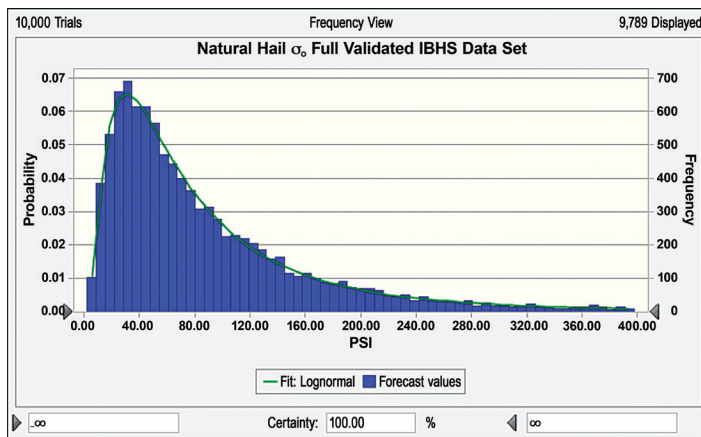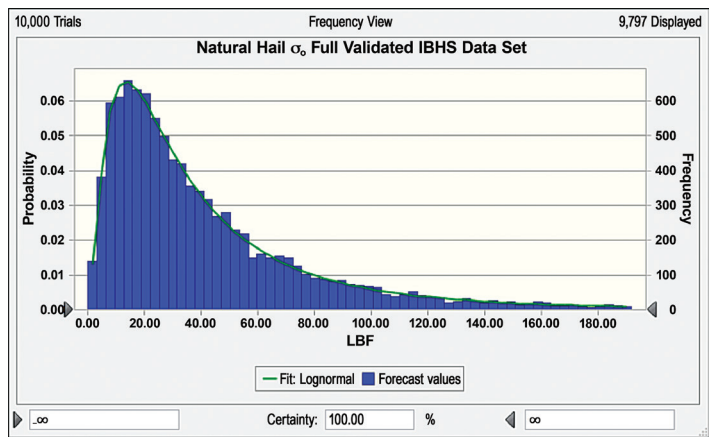

Figure 13A – Natural hail average F_o.
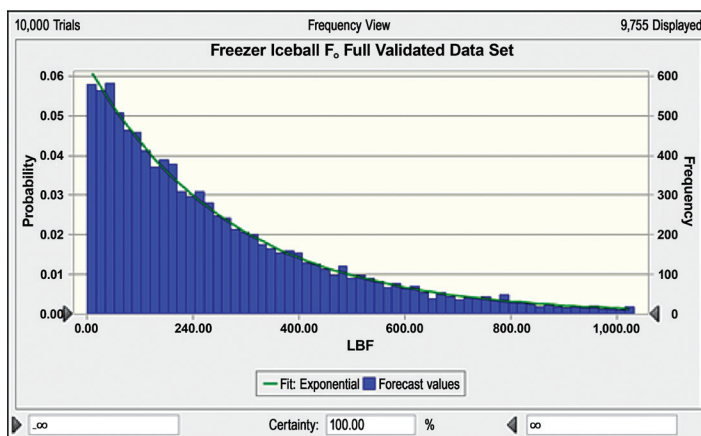

Figure 13B – Natural hail average σ_c.

Figure 13C – Iceball average F_o.
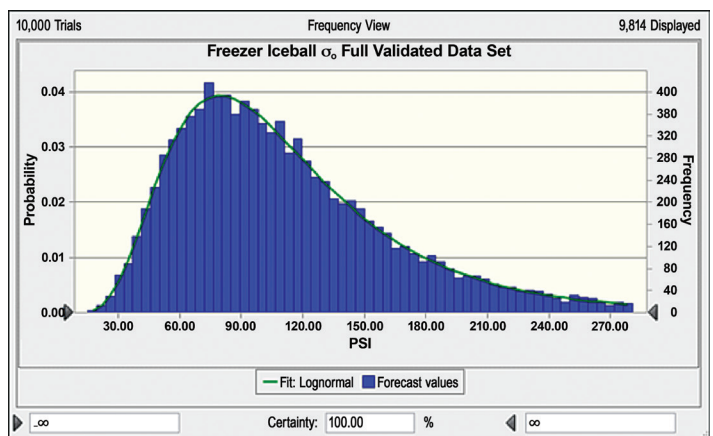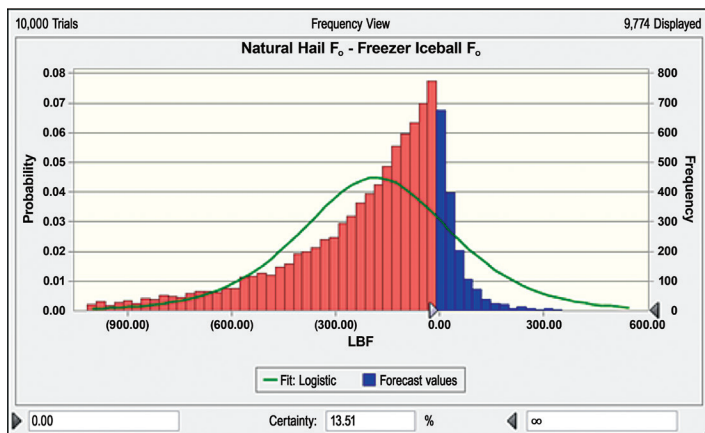

Figure 13D – Iceball average σ_o.
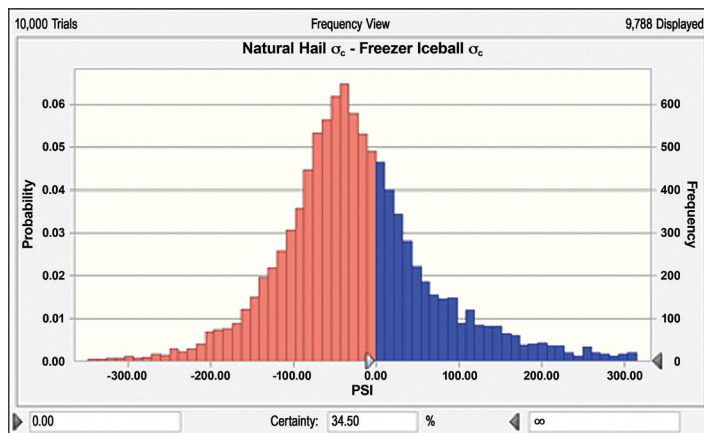
Figure 14A – Hail probability >= iceball F_o.

Figure 14B – Hail probability >= iceball σ_c.

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Charles A. Doswell III. *Severe Convective*

Measured	Model		Anova: Single Factor		α	0.05						
28.28171642	22.83646559									LSD	30.24516	
29.47628	29.33129432		SUMMARY							HSD	30.24486	
37.843709	52.83414079		*Groups*	*Count*	*Sum*	*Average*	*Variance*			Scheffe	124.704	
55.11070555	61.86450858		Measured	18	1415.34	78.63002	2487.426			Post Hoc	Measured	
66.67345899	68.45066266		Model	18	1367.426	75.96814	1499.446			Model	2.661885	
91.71079858	85.9438233									Colored cells have signficant mean differences		
102.463496	97.8063072											
106.5000268	128.5445801		ANOVA			Cannot Reject Null Hypothesis because p > 0.05 (Means are the same)						
189.61	136.1014396		*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-Value*	*F crit*			
28.28171642	22.83646559		Between Groups	63.77071	1	63.77071	0.03199	0.859	4.130018			
29.47628	29.33129432		Within Groups	67776.82	34	1993.436						
37.843709	52.83414079											
55.11070555	61.86450858		Total	67840.59	35							
66.67345899	68.45066266											
91.71079858	85.9438233											
102.463496	97.8063072											
106.5000268	128.5445801											
189.61	136.1014396											

Table 4 – ANOVA for natural hail and iceballs modeled as natural hail F_o.

Measured	Mean
264.888565	128.6162517
97.14518358	90.07386076
65.96082252	135.1153394
57.46370929	63.09071812
58.42781866	49.02059056
45.04125918	39.30160604
39.29400641	37.71469509
31.59819961	48.01990126
34.80970514	36.31716211
264.888565	128.6162517
97.14518358	90.07386076
65.96082252	135.1153394
57.46370929	63.09071812
58.42781866	49.02059056
45.04125918	39.30160604
39.29400641	37.71469509
31.59819961	48.01990126
34.80970514	36.31716211

Anova: Single Factor α 0.05

					LSD	38.5342
SUMMARY					HSD	38.53383
Groups	*Count*	*Sum*	*Average*	*Variance*	Scheffe	158.8806
Measured	18	1389.259	77.18103	5040.065	Post Hoc	Measured
Mean	18	1254.54	69.69668	1431.561	Mean	7.484349

Colored cells have signficant mean differences

ANOVA   Cannot Reject Null Hypothesis because p > 0.05 (Means are the same)

Source of Variation	*SS*	*df*	*MS*	*F*	*P-Value*	*F crit*
Between Groups	504.1394	1	504.1394	0.1558	0.696	4.130018
Within Groups	110017.6	34	3235.813			
Total	110521.8	35				

Measured σ_c - Modeled σ_c

Table 5 – ANOVA for natural hail and iceballs modeled as natural hail σ_c.

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